

0.5 $\overline{AB \wedge AC}(3,3,3)$: $\overline{AC}(1,-1,0)$ $\overline{AB}(0,-3,3)$: - (1)
 : (ABC) $\overline{AB \wedge AC}$: -

$$M(x, y, z) \in (ABC) \Leftrightarrow \overline{AM} \cdot (\overline{AB} \wedge \overline{AC}) = 0$$

$$\Leftrightarrow 3 \cdot (x-1) + 3 \cdot (y-2) + 3 \cdot (z+2) = 0$$

$$\Leftrightarrow 3 \cdot (x+y+z-1) = 0$$

0.5 $x+y+z-1=0$: (ABC)
 $d = \frac{|1+1+1-1|}{\sqrt{1^2+1^2+1^2}} = \frac{2}{\sqrt{3}}$: (ABC) $\Omega(1,1,1)$ - (2)

0.5 H (S) (ABC) : $d=R$:
 $\begin{cases} x=1+t \\ y=1+t \\ z=1+t \end{cases} (t \in \mathbb{R})$: (ΩH) (ABC)

0.75 $H(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$: $t = \frac{-2}{3}$: $\begin{cases} x=1+t \\ y=1+t \\ z=1+t \\ x+y+z-1=0 \end{cases}$:
 (ABC) $M(a,b,c)$ -

$$\begin{cases} a+b+c-1=0 \\ (a-1)^2+(b-1)^2+(c-1)^2 \geq \frac{4}{3} \end{cases} : \begin{cases} a+b+c-1=0 \\ d(M, \Omega) \geq \frac{2}{\sqrt{3}} \end{cases} :$$

$$\begin{cases} a+b+c=1 \\ a^2+b^2+c^2-2+3 \geq \frac{4}{3} \end{cases} : \begin{cases} a+b+c=1 \\ a^2+b^2+c^2-2(a+b+c)+3 \geq \frac{4}{3} \end{cases}$$

0.75 $a^2+b^2+c^2 \geq \frac{1}{3}$:

$$v_{n+1} = u_{n+2} - \frac{1}{5}u_{n+1} = (\frac{2}{5}u_{n+1} - \frac{1}{25}u_n) - \frac{1}{5}u_{n+1}$$

$$= \frac{1}{5}u_{n+1} - \frac{1}{25}u_n = \frac{1}{5}(u_{n+1} - \frac{1}{5}u_n)$$

: \mathbb{N} n (1)

$v_{n+1} = \frac{1}{5}v_n$:

0.5 $v_0 = u_1 - \frac{1}{5}u_0 = 1$: $q = \frac{1}{5}$ (v_n) :

0.25 $\forall n \in \mathbb{N} : v_n = (\frac{1}{5})^n$:

$w_{n+1} = 5^{n+1}u_{n+1} = 5^{n+1}(v_n + \frac{1}{5}u_n) = 5^{n+1} \cdot \frac{1}{5^n} + 5^n u_n$: \mathbb{N} n - (2)

0.25 $w_0 = 0$: $r = 5$ (w_n) : $w_{n+1} = 5 + w_n$:

$(\forall n \in \mathbb{N} : u_n = \frac{w_n}{5^n})$: 0.25 $(\forall n \in \mathbb{N} : w_n = 5 \cdot n)$: -

$$E(X) = 0 \cdot \frac{11}{50} + 1 \cdot \frac{30}{50} + 2 \cdot \frac{9}{50} = \frac{48}{50} = \frac{24}{25} \quad ; \quad X \quad (2)$$

$$0.25 \quad \boxed{E(X) = \frac{24}{25}} \quad :$$

$$\delta = \sqrt{2} \cdot (1-i) \quad \Delta \quad 0.25 \quad \Delta = 4 - 4(1+i) = -4i \quad ; \quad (1)$$

$$(\text{Im}(z_1) > 0 :) \quad z_2 = \frac{-2 + \sqrt{2} \cdot (1-i)}{2} \quad z_1 = \frac{-2 - \sqrt{2} \cdot (1-i)}{2} \quad ;$$

$$0.25 \quad \boxed{z_2 = -1 + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i} \quad 0.25 \quad \boxed{z_1 = -1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i} \quad ;$$

$$-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = -\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) + i \sin\left(\pi - \frac{\pi}{4}\right) \quad ; \quad - \quad (2)$$

$$0.5 \quad \boxed{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) = \left[1, \frac{3\pi}{4}\right]} \quad ;$$

$$z_{M_1} - z_A = -1 - \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i + 1 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = z_B - z_O \quad ;$$

$$0.25 \quad \boxed{\overrightarrow{AM_1} = \overrightarrow{OB}} \quad ;$$

$$0.25 \quad [M_1 M_2] \quad A \quad ; \quad \frac{z_{M_1} + z_{M_2}}{2} = \frac{-2}{2} = -1 = z_A \quad ;$$

$$OB = OA = 1 \quad ; \quad AOBM \quad ; \quad \overrightarrow{AM_1} = \overrightarrow{OB} \quad ; \quad 0.5 \quad ; \quad AOBM \quad ;$$

$$(\overrightarrow{e_1}, \overrightarrow{OM_1}) \equiv (\overrightarrow{e_1}, \overrightarrow{OB}) + (\overrightarrow{OB}, \overrightarrow{OM_1}) \equiv \frac{3\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4} + \frac{\pi}{8} [2\pi] \quad ;$$

$$0.5 \quad \boxed{\text{Arg}z_1 \equiv \frac{7\pi}{8} [2\pi]} \quad ; \quad (\overrightarrow{e_1}, \overrightarrow{OM_1}) \equiv \frac{7\pi}{8} [2\pi] \quad ;$$

$$r = 1: \quad r^2 - 2r + 1 = 0: \quad y'' - 2y' + y = 0: \quad (1) \quad 0.25$$

$$0.5 \quad (a, b) \in \mathbb{R}^2 \quad \boxed{y: x \mapsto (ax+b)e^x} \quad ; \quad \forall x \in \mathbb{R}: (y_0'(x) = a \quad y_0''(x) = 0) \quad ; \quad - \quad (2)$$

$$y_0 \Leftrightarrow \forall x \in \mathbb{R}: y_0''(x) - 2y_0'(x) + y_0(x) = x - 1$$

$$\Leftrightarrow \forall x \in \mathbb{R}: -2a + ax + b = x - 1$$

$$\Leftrightarrow \begin{cases} a = 1 \\ b - 2a = -1 \end{cases} \quad \Leftrightarrow \begin{cases} a = 1 \\ b = 1 \end{cases}$$

$$0.25 \quad (E) \quad \boxed{y_0: x \mapsto x + 1} \quad ;$$

$$0.25 \quad \boxed{(a, b) \in \mathbb{R}^2 : y: x \mapsto (ax+b)e^x + x + 1}$$

$$\forall x \in \mathbb{R}: \begin{cases} h(x) = (ax+b)e^x + x + 1 \\ h'(x) = (ax+a+b)e^x + 1 \end{cases} \quad ; \quad -$$

$$\begin{cases} h(0) = 0 \Leftrightarrow b+1=0 \Leftrightarrow b=-1 \\ h'(0) = 1 \Leftrightarrow a+b+1=1 \Leftrightarrow b=-a \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=-1 \end{cases} :$$

0.5 $\boxed{\forall x \in \mathbb{R} : h(x) = (x-1)e^x + x + 1} :$

$\forall x \in \mathbb{R} : g'(x) = e^x + (x-1)e^x + 1 :$ - (3)

0.5 $\boxed{\forall x \in \mathbb{R} : g'(x) = xe^x + 1} :$

0.25 $\boxed{[0, +\infty[\quad g}$

$\forall x \in [0, +\infty[: xe^x + 1 > 0 :$

$[0, +\infty[\quad g \quad g(0) : [0, +\infty[\quad g \quad -$
 $\forall x \in [0, +\infty[: g(x) \geq g(0) :$

0.25 $\boxed{\forall x \in [0, +\infty[: g(x) \geq 0} : g(0) = 0 :$
 $: \quad -x \in \mathbb{R}^* \quad \mathbb{R}^* \quad x \quad (1 - II$

$$f(-x) = \frac{-xe^{-x}}{(e^{-x}-1)^2} = \frac{\frac{-x}{e^x}}{\left(\frac{1}{e^x}-1\right)^2} = -\frac{\frac{x}{e^x}}{\left(\frac{1-e^x}{e^x}\right)^2} = -\frac{x}{e^x} \cdot \frac{e^{2x}}{(e^x-1)^2} = -\frac{xe^x}{(e^x-1)^2}$$

0.5 $\boxed{f} \quad \boxed{\forall x \in \mathbb{R}^* : f(-x) = -f(x)} :$

05 $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{xe^x}{(e^x-1)^2} = \lim_{x \rightarrow 0^+} \frac{x^2 \cdot e^x}{x \cdot (e^x-1)^2} = \lim_{x \rightarrow 0^+} \frac{e^x}{x} \cdot \frac{1}{(e^x-1)^2} = +\infty :$ - (2)

0.25 $\lim_{x \rightarrow 0} \frac{e^x}{x} = +\infty$

$\lim_{x \rightarrow 0^+} \frac{e^x-1}{x} = 1 :$

0.25 $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{x}{e^x \cdot (1-e^{-x})^2} = \lim_{x \rightarrow +\infty} \frac{x}{e^x} \cdot \frac{1}{(1-e^{-x})^2} = 0$ -

0.25 $y = 0 \quad \lim_{x \rightarrow +\infty} (1-e^{-x})^2 = 1 \quad \lim_{x \rightarrow +\infty} \frac{e^x}{x} = +\infty :$
 $: \quad - \quad (3$

$$\forall x \in \mathbb{R}^* : f'(x) = \frac{(e^x + xe^x)(e^x-1)^2 - 2xe^{2x}(e^x-1)}{(e^x-1)^4} = \frac{(e^x-1)[(e^x-1)(e^x + xe^x) - 2xe^{2x}]}{(e^x-1)^4}$$

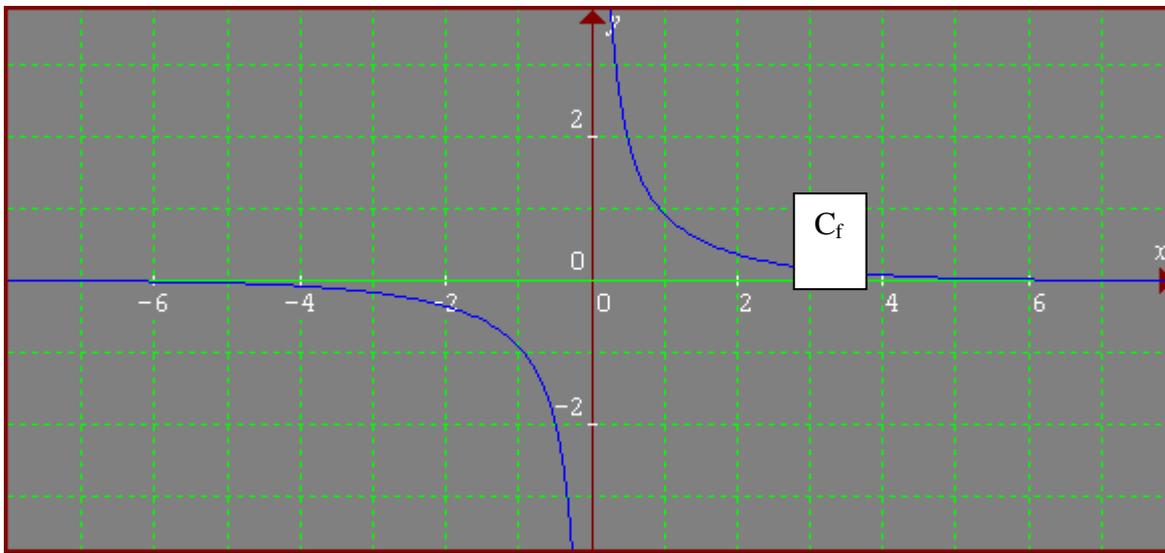
$$= \frac{e^{2x} + xe^{2x} - e^x - xe^x - 2xe^{2x}}{(e^x-1)^3} = e^x \cdot \frac{e^x - xe^x - x - 1}{(e^x-1)^3}$$

0.75 $\boxed{\forall x \in \mathbb{R}^* : f'(x) = -\frac{e^x}{(e^x-1)^3} \cdot g(x)} :$

$: \quad f'(x) < 0 \quad : \quad g(x) > 0 \quad e^x > 0 \quad e^x - 1 > 0 \quad : \quad]0, +\infty[\quad :$
 0.5 :

x	0	+	+
f'(x)		-	
f(x)	+		0

0.5 : f _____ (4)



$$\int_2^3 \frac{1}{t(t-1)} dt = \int_2^3 \left(\frac{1}{t-1} - \frac{1}{t} \right) dt = [\ln(t-1) - \ln t]_2^3 \quad : \quad - \quad (5)$$

$$= \left[\ln\left(\frac{t-1}{t}\right) \right]_2^3 = \ln\left(\frac{2}{3}\right) - \ln\left(\frac{1}{2}\right) = \ln 2 - \ln 3 + \ln 2 = \boxed{2\ln 2 - \ln 3} \quad 0.5$$

$$f(x) = \frac{t \cdot \ln t}{(t-1)^2} \quad dx = \frac{1}{t} dt \quad x = \ln t \quad : \quad t = e^x \quad : \quad -$$

$$\begin{cases} x = 2 \Rightarrow t = \ln 2 \\ x = 3 \Rightarrow t = \ln 3 \end{cases} \quad :$$

$$0.5 \quad \int_{\ln 2}^{\ln 3} f(x) dx = \int \frac{t \cdot \ln t}{(t-1)^2} \cdot \frac{1}{t} dt = \int_2^3 \frac{\ln t}{(t-1)^2} dt \quad :$$

$$\begin{cases} u'(t) = \frac{1}{t} & \begin{cases} u(t) = \ln t \\ v'(t) = \frac{1}{(t-1)^2} \end{cases} \\ v(t) = \frac{-1}{t-1} \end{cases} \quad : \quad - \quad (6)$$

$$\begin{aligned} \int_2^3 \frac{\ln t}{(t-1)^2} dt &= \left[\frac{-\ln t}{t-1} \right]_2^3 - \int_2^3 \frac{-1}{t(t-1)} dt \quad : \\ &= \frac{-\ln 3}{2} + \ln 2 + 2\ln 2 - \ln 3 \end{aligned}$$

$$0.5 \quad \boxed{\int_2^3 \frac{\ln t}{(t-1)^2} dt = 3\ln 2 - \frac{3}{2}\ln 3} \quad :$$

$$A = \int_{\ln 2}^{\ln 3} f(x) dx \quad : \quad [2, 3] \quad f \quad -$$

$$0.5 \quad \boxed{A = 0.45 \text{ u.a}} \quad . \quad \boxed{A = 3\ln 2 - \frac{3}{2}\ln 3} \quad :$$

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